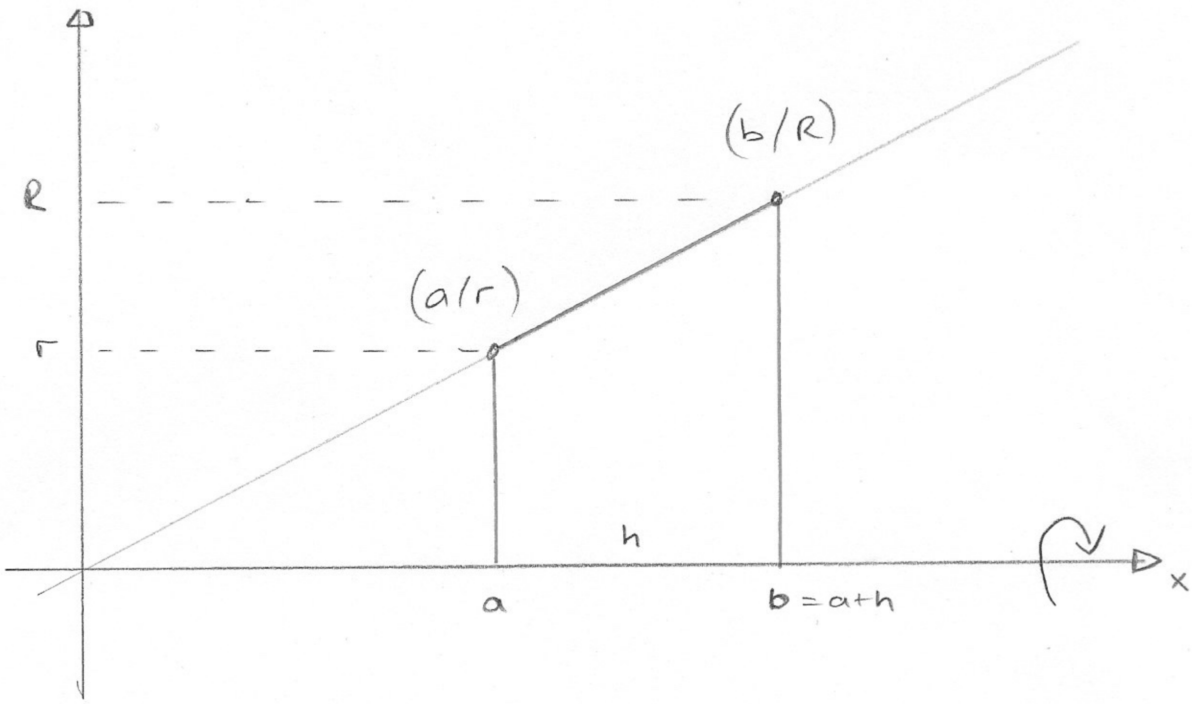
A8



$$f(x) = mx \qquad m = \frac{\Delta y}{\Delta x} = \frac{R-r}{b-a}$$

$$= \frac{R-r}{a+h-a} = \frac{R-r}{h}$$

a) Der Kegelstumpf ensteht durch Rotation eines Geradenstücks um die x-Achse. A8

b) $f(x) = mx$ Ansatz $P_1(a/r)$ $P_2(b/R)$ $b = a+h$

$$m = \frac{\Delta y}{\Delta x} = \frac{R-r}{b-a} = \frac{R-r}{a+h-a} = \frac{R-r}{h}$$

$$f(x) = \frac{R-r}{h} x$$

c) $f(a) = \frac{R-r}{h} a = r \quad \Rightarrow a = \frac{rh}{R-r}$

$f(b) = \frac{R-r}{h} b = R \quad \Rightarrow b = \frac{Rh}{R-r}$

d)

$$V = \pi \int_a^b f^2(x)\,dx = \pi \int_{\frac{rh}{R-r}}^{\frac{Rh}{R-r}} \frac{(R-r)^2}{h^2} x^2\,dx$$

$$= \pi \left[\frac{(R-r)^2}{3h^2} x^3 \right]_{\frac{rh}{R-r}}^{\frac{Rh}{R-r}} =$$

$$= \pi \left(\frac{(R-r)^2}{3h^2} \frac{R^3h^3}{(R-r)^3} - \frac{(R-r)^2}{3h^2} \frac{r^3h^3}{(R-r)^3} \right)$$

$$= \frac{\pi h}{3} \left(\frac{R^3}{R-r} - \frac{r^3}{R-r} \right) = \frac{\pi}{3} h \left(\frac{R^3-r^3}{R-r} \right)$$

$\frac{R^3-r^3}{R-r} = ?$ Polynomdivision

$$(R^3 + 0R^2 + 0R - r^3) : (R-r) = R^2 + rR + r^2$$

$$\begin{array}{l} R^3 - rR^2 \\ \hline rR^2 + 0R \\ rR^2 - r^2R \\ \hline r^2R - r^3 \\ r^2R - r^3 \\ \hline 0 \end{array}$$

$$\Rightarrow V = \frac{\pi}{3} h \left(R^2 + rR + r^2 \right)$$