

17c

$$\frac{5x^2 - 24x + 36}{x(x-3)^2} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$$5x^2 - 24x + 36 = A(x-3)^2 + B(x-3)x + Cx$$

$x=3$ einsetzen

$$5 \cdot 9 - 24 \cdot 3 + 36 = 3C$$

$$9 = 3C \quad \leadsto \quad C = 3$$

$x=0$ einsetzen

$$36 = A \cdot 9 \quad \leadsto \quad A = 4$$

$$C = 3 \quad A = 4 \quad x = 1$$

$$5 - 24 + 36 = 4 \cdot (-2)^2 + B(-2) \cdot 1 + 3 \cdot 1$$

$$17 = 16 - 2B + 3$$

$$-2 = -2B \quad \leadsto \quad B = 1$$

$$\int f(x) dx = \int \frac{4}{x} dx + \int \frac{1}{x-3} dx + \int \frac{3}{(x-3)^2} dx$$

$$= 4 \ln(|x|) + \ln(|x-3|) + \int 3(x-3)^{-2} dx$$

$$= 4 \ln(|x|) + \ln(|x-3|) - 3(x-3)^{-1} + C$$

18a)

$$\int \frac{2x^2 + 5x + 2}{x^3 + x} dx$$

$$\frac{2x^2 + 5x + 2}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$$

$$2x^2 + 5x + 2 = A(x^2 + 1) + (Bx + C)x$$

$$2x^2 + 5x + 2 = Ax^2 + A + Bx^2 + Cx$$

$$2x^2 + 5x + 2 = (A + B)x^2 + Cx + A$$

$$A + B = 2$$

$$C = 5$$

$$A = 2 \Rightarrow 2 + B = 2 \Rightarrow B = 0$$

$$\int \frac{2x^2 + 5x + 2}{x^3 + x} dx = \int \frac{2}{x} dx + \int \frac{5}{x^2 + 1} dx$$

$$= 2 \ln(|x|) + 5 \arctan(x) + C$$

